

BOOK REVIEW

Scaling Limits and Models in Physical Processes. By C. CERCIGNANI & D. H. SATTINGER. DMV Seminar Band 28. Birkhäuser, 1998. 190 pp. ISBN 3 7643 5985 4 (Basel), 0 8176 5985 4 (Boston). DM 58.

When physical problems contain several small parameters, meaningful asymptotics and corresponding mathematical models appear under definite scaling relations between these parameters. A typical example is the Cauchy–Poisson problem for surface waves in a horizontal layer. The problem contains two small parameters:

$$\delta_1 = \frac{a}{\lambda}, \quad \delta_2 = \frac{a}{h_0}, \quad (1)$$

where a is the wave amplitude, λ the wavelength, and h_0 an undisturbed layer thickness. Asymptotics leading to the Korteweg–de Vries equation is obtained when $\delta_1 = O(\delta_2^{3/2})$.

The idea of obtaining mathematical models by proper scaling of small parameters in multiparametric nonlinear problems is presented and illustrated in detail in the volume under review by two models of different and important phenomena. The volume is written by two very different but equally outstanding specialists, each presenting the models in his own style.

I can compare this volume with a concerto of two different but equally outstanding soloists, like, for instance, Richter and Menukhin. The two parts of the book are joined by a common general idea and also include two practically isomorphic prefaces (expressing well deserved gratitude to Professor Dr W. Jäger who organized the series of lectures which led to this volume). Like in concertos the reader has the feeling of a remarkable competition between the two authors, and this is very good—the more of such competitions there are, the better it will serve the applied mathematics and fluid mechanics communities.

The first part, *Scaling and Mathematical Models in Kinetic Theory*, is written by Carlo Cercignani. It consists of two chapters: 1. Boltzmann equation and gas surface interaction, and 2. Perturbation methods for the Boltzmann equation. The second part, *Scaling, Mathematical Modelling, and Integrable Systems*, belongs to David Sattinger. It is divided into eight chapters: 1. Dispersion, 2. The nonlinear Schrödinger equation, 3. The Korteweg–de Vries equation, 4. Isospectral deformations, 5. Inverse scattering theory, 6. Variational methods, 7. Weak and strong nonlinearities, and 8. Numerical methods.

I strongly recommend this compact (190 pages) volume to all mathematical and technical libraries and even for small individual libraries of teachers, advanced students, and researchers in applied mathematics, physics, and engineering science. It will be good to have this volume on your own bookshelf. It presents more than its special subjects: basic concepts of kinetic theory of gases and general ideas of modelling weakly nonlinear dispersive phenomena—the reader achieves the general feeling of nonlinear nonequilibrium phenomena. The historical comments of both authors are concise but also rich in content.

The reviewer takes the liberty of making a couple of minor comments. Both

parts are announced as introductory, and I will strongly recommend the book to my students. However, as far as the first part is concerned, I would recommend students start their reading from sections 2.1–2.3 of the second chapter, and then go back to the beginning. It is in these sections, and not at the beginning, that a clear general explanation of the whole subject is given, including technical applications and interesting historical comments, and to start from these sections will be, I think, much better for readers' inspiration.

I would also like to mention the exposition in the second part of the Benjamin–Bona–Mahoney equation

$$\partial_t h + u \partial_x u + \partial_{xxx}^3 u = 0, \quad (2)$$

which is, so-to-speak, competitive with the Korteweg–de Vries equation. As it is presented on pages 110 and 164, it leaves the reader, and especially the beginner, puzzled. We read (page 110), 'In other words both the KdV and BBM approximations are accurate for a finite time scale ... and they are equally valid models over that time interval'. But the experiments in particular mentioned by the author give something in between. So, what is the author's advice to a beginner? This is in fact a serious problem.

I repeat that these and some other comments are very minor ones and by no means reduce the excellent impression of the book.

G. I. BARENBLATT